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# Almost $n$-ary Ideals of $n$-ary Semigroups

**Pattarawan Singavananda**[1], **Ronnason Chinram**[2]

[1]Mathematics Program
Faculty of Science and Technology
Songkhla Rajabhat University
Songkhla 90000, Thailand

[2]Division of Computational Science
Faculty of Science
Prince of Songkla University
Hat Yai, Songkhla 90110, Thailand

email: pattarawan.pe@skru.ac.th, ronnason.c@psu.ac.th

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**Abstract**

In this paper, we introduce the concepts of almost $i$-ideals and almost ideals in $n$-ary semigroups and investigate their properties. Some of the results presented here extend existing findings on almost ideals in both semigroups and ternary semigroups.

## 1 Introduction and Preliminaries

The study of algebraic systems involving $n$-ary operations was first initiated by Kasner [4] in 1904. It is noteworthy that semigroups and ternary semigroups constitute special cases of $n$-ary semigroups for $n = 2$ and $n = 3$, respectively. Almost ideals on semigroups were first studied by Grosek and Satko [3] in 1980. Later, almost ideals on ternary semigroups were examined in detail in [7]. Recently, almost $n$-ary subsemigroups and their fuzzifications

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of $n$-ary semigroups were studied in [1]. In this section, we present some preliminary notions of $n$-ary semigroups, based on the frameworks provided in [1, 2, 5, 6]. A nonempty set $A$ together with an $n$-ary operation given by $f : A^n \to A$, where $n \geq 2$, is called an *n-ary groupoid* and is denoted by $(A, f)$ (or in short $A$). For $i \leq j$, the sequence of elements $a_i, a_{i+1}, \ldots, a_j$ in $A$ is denoted by $a_i^j$. In the case $j < i$, it is the empty symbol. If $a_{i+1} = a_{i+2} = \cdots = a_{i+t} = a$, then we will write $a^t$ instead of $a_{i+1}^{i+t}$. In this convention, we get that $f(a_1, a_2, \ldots, a_n) = f(a_1^n)$ and

$$f(a_1, \ldots, a_i, \underbrace{a \ldots, a}_{t-times}, a_{i+t+1}, \ldots, a_n) = f(a_1^i, a^t, a_{i+t+1}^n).$$

An $n$-ary groupoid $A$ is called $(i, j)$-*associative* if for all $a_1, a_2, \ldots, a_{2n-1} \in A$, $f(a_1^{i-1}, f(a_i^{n+i-1}), a_{n+i}^{2n-1}) = f(a_1^{j-1}, f(a_j^{n+j-1}), a_{n+j}^{2n-1})$. An $n$-ary groupoid $A$ is called *associative* if the above identity holds for every $1 \leq i \leq j \leq n$. In this case, $A$ is called an *n-ary semigroup*.
Let $A$ be an $n$-ary semigroup $A$. For $x_1, \ldots, x_{i-1}, x_{i+1}, ..., x_n \in A$ and a nonempty subset $S$ of $A$, let $f(x_1^{i-1}, S, x_{i+1}^n) = \{f(x_1^{i-1}, a, x_{i+1}^n) \mid a \in S\}$. A nonempty subset $I$ of $A$ is called an *i-ideal* of $A$ if $f(x_1^{i-1}, I, x_{i+1}^n) \subseteq I$ for every $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n \in A$. A nonempty subset $I$ of $S$ is called an *ideal* of $A$ if $I$ is an $i$-ideal for every $1 \leq i \leq n$.
In this paper, we generalize the findings presented in [3] and [7]. We introduce the concepts of almost $i$-ideals and almost ideals of $n$-ary semigroups, and present their properties.

# 2 Main Results

We commence by formally introducing the notions of almost $i$-ideals and almost ideals within the framework of $n$-ary semigroups.

**Definition 2.1.** A nonempty subset $I$ of an $n$-ary semigroup $A$ is called an *almost i-ideal* of $A$ if for every $b \in A$, $f(b^{i-1}, I, b^{n-i}) \cap I \neq \emptyset$. If $I$ is an almost $i$-ideal of $A$ for all $1 \leq i \leq n$, then $I$ is called an *almost ideal* of $A$.

Every $i$-ideal of an $n$-ary semigroup $A$ is clearly an almost $i$-ideal of $A$. However, in general, an almost $i$-ideal of $A$ need not be an $i$-ideal. Illustrative examples in the cases of semigroups and ternary semigroups can be found in Example 1 of [3] and Example 3.2 of [7], respectively.

**Theorem 2.2.** *Let $I$ be an almost $i$-ideal of an $n$-ary semigroup $A$. Then every subset $J \subseteq A$ containing $I$ (i.e.,$I \subseteq J$) is also an almost $i$-ideal of $A$.*

*Proof.* Let $I$ be an almost $i$-ideal of $A$, and let $J \subseteq A$ be such that $I \subseteq J$, with $b \in A$. Since $f(b^{i-1}, I, b^{n-i}) \cap I \neq \emptyset$ and $f(b^{i-1}, I, b^{n-i}) \cap I \subseteq f(b^{i-1}, J, b^{n-i}) \cap J$, it follows that $f(b^{i-1}, J, b^{n-i}) \cap J \neq \emptyset$. Therefore, $J$ is also an almost $i$-ideal of $A$. □

The following corollary follows directly from Theorem 2.2.

**Corollary 2.3.** *Let $A$ be an $n$-ary semigroup.*

*(1) The union of almost $i$-ideals of $A$ is also an almost $i$-ideal of $A$.*

*(2) The union of almost ideals of $A$ is also an almost ideal of $A$.*

However, the intersection of almost ideals of $A$ need not be an almost ideal of $A$. Counterexamples can be found in the cases of semigroups and ternary semigroups, specifically in Example 1 of [3] and Example 3.3 of [7], respectively.

**Theorem 2.4.** *Let $A$ be an $n$-ary semigroup such that $|A| > 1$. Then $A$ has no proper almost $i$-ideals if and only if for every $a \in A$, there exists an element $b_a \in A$ such that $f(b_a^{i-1}, A \setminus \{a\}, b_a^{n-i}) = \{a\}$.*

*Proof.* Suppose that an $n$-ary semigroup $A$ has no proper almost $i$-ideals, and let $a \in A$. Then $A \setminus \{a\}$ is not an almost $i$-ideal of $A$. So there exists $b_a \in A$ such that $f(b_a^{i-1}, A \setminus \{a\}, b_a^{n-i}) \cap (A \setminus \{a\}) = \emptyset$. It follows that $f(b_a^{i-1}, A \setminus \{a\}, b_a^{n-i}) = \{a\}$. Conversely, let $a \in A$. So there exists $b_a \in A$ such that $f(b_a^{i-1}, A \setminus \{a\}, b_a^{n-i}) = \{a\}$. Then clearly, $f(b_a^{i-1}, A \setminus \{a\}, b_a^{n-i}) \cap (A \setminus \{a\}) = \emptyset$, which implies that $A \setminus \{a\}$ is not an almost $i$-ideal. Consequently, $A$ containing no proper almost $i$-ideals. □

**Theorem 2.5.** *Let $A$ be an $n$-ary semigroup such that $|A| > 1$, and let $a \in A$. If $A \setminus \{a\}$ is not an almost $i$-ideal of $A$, then either $f(a^n) = a$ or $f(f(f(a^n), a^{n-1}), a^{n-1}) = a$.*

*Proof.* Assume that $A \setminus \{a\}$ is not an almost $i$-ideal of $A$. By Theorem 2.4, there exists $b_a \in A$ such that $f(b_a^{i-1}, A \setminus \{a\}, b_a^{n-i}) = \{a\}$. Suppose that $f(a^n) \neq a$. Thus $f(b_a^{i-1}, f(a^n), b_a^{n-i}) = a$.

Case 1: Suppose that $b_a = a$. Then $f(f(a^n), a^{n-1}) = f(a^{i-1}, f(a^n), a^{n-i}) = a$. Thus $f(f(f(a^n), a^{n-1}), a^{n-1}) = a$.

Case 2: Suppose that $b_a \neq a$. Then $f(b_a^n) = f(b_a^{i-1}, b_a, b_a^{n-i}) = a$.

Subcase 2.1: If $f(b_a^{i-1}, a, b_a^{n-i}) = a$, then $f(b_a^{i-1}, f(a^n), b_a^{n-i}) = f(a^n) \neq a$, which is a contradiction.

Subcase 2.2: If $f(b_a^{i-1}, a, b_a^{n-i}) \neq a$, then $f(b_a^{i-1}, f(b_a^{i-1}, a, b_a^{n-i}), b_a^{n-i}) = a$. Since $f(b_a^n) = a$, it follows that $f(f(f(a^n), a^{n-1}), a^{n-1}) = a$. □

Note that for $n = 2$, Theorem 2.5 is Lemma 2 in [3] and for $n = 3$, Theorem 2.5 is Theorem 3.3 in [7].

# 3 Conclusion

In this paper, we introduced and studied the concept of almost $i$-ideals and almost ideals in $n$-ary semigroups, extending previous work on semigroups and ternary semigroups. We established fundamental properties, including closure under set inclusion but not under intersection, as well as characterizations for the existence of proper almost $i$-ideals. Additionally, we examined structural conditions linking the failure of certain subsets to be almost $i$-ideals with specific functional identities. These results contribute to the broader understanding of $n$-ary algebraic structures and provide a basis for future exploration in generalized algebraic frameworks.

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