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On Essential Subsemirings of Semirings

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Abstract

In this paper, we define essential subsemirings of semirings and investigate their properties. Moreover, we characterize when a subsemiring of semirings of all non-negative integers under the usual addition and multiplication is essential.

1 Introduction and Preliminaries

A semiring as an algebraic structure is a generalization of rings, dropping the requirement that each element must have an additive inverse. By a semiring, we mean a nonempty set R endowed with two binary operations called the addition $+$ and multiplication $\cdot$ satisfying $(R,+)$ is a commutative semigroup, $(R,\cdot)$ is a semigroup, and the multiplication distributes over the addition both from the left and from the right. An additive subsemigroup A

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of a semiring R is a subsemiring of R if $A^2 \subseteq A$, and an ideal of R if $RA \subseteq A$ and $AR \subseteq A$. A proper ideal I of a ring R is essential if I has nonzero intersection with each nonzero ideal of R ([3]). Similar to essential ideals in rings, essential ideals of semirings were studied in [2]. Later, Pawar [5] defined weak essential ideals in semirings and investigated their properties. Next, essential ideals in semigroups were defined and studied in [1]. In 2023, Panpetch, Muangngao and Gaketem [4] defined essential subsemigroups of semigroups and, as a result, our goal is to define essential subsemirings of semirings.

2 Main results

First, we define essential subsemirings of semirings as follows:

Definition 2.1. *A subsemiring A of a semiring R is called* essential *if $A \cap S \neq \emptyset$ for every subsemiring S of R.*

Theorem 2.2. *Let A and B be any two essential subsemirings of a semiring R. Then $A \cap B$ is also an essential subsemiring of R.*

Proof. By definition of essential subsemirings, $A \cap B \neq \emptyset$ and $A \cap B$ is clearly a subsemiring of R. Let S be any subsemiring of R. Then $B \cap S \neq \emptyset$ because B is essential. Thus $B \cap S$ is also a subsemiring of R. Since $B \cap S$ is a subsemiring of R and A is essential, $A \cap (B \cap S) \neq \emptyset$. This implies that $(A \cap B) \cap S \neq \emptyset$ and we conclude that $A \cap B$ is an essential subsemiring of R. □

Theorem 2.3. *Let A and B be any two subsemirings of a semiring R. If $A \subseteq B$ and A is essential, then B is also essential.*

Proof. Let S be any subsemiring of R. Since A is essential, $A \cap S \neq \emptyset$. Then $B \cap S \neq \emptyset$. Hence B is essential. □

Since the union of two subsemirings of a semiring R need not be a subsemiring of R, the union of two essential subsemirings of R need not be an essential subsemiring of R.

Corollary 2.4. *Let A and B be essential subsemirings of a semiring R. If $A \cup B$ is a subsemiring of R, then $A \cup B$ is essential.*

We consider a semiring $\mathbb{N}_0$ of the set of all non-negative integers under the usual addition and multiplication. We have that $\{0\}$ and $\mathbb{N}$ are subsemirings of $\mathbb{N}_0$. Since $\{0\} \cap \mathbb{N} = \emptyset$, $\{0\}$ and $\mathbb{N}$ are not essential. Next, we characterize when a subsemiring of $\mathbb{N}_0$ is essential.

Theorem 2.5. *A subsemiring A of a semiring $\mathbb{N}_0$ is essential if and only if $0 \in A$ and $A \neq \{0\}$.*

Proof. Let A be a subsemiring of $\mathbb{N}_0$. Assume that A is essential. Since $\{0\}$ is a subsemiring of $\mathbb{N}_0$, $A \cap \{0\} \neq \emptyset$. Thus $0 \in A$. Since $\mathbb{N}$ is a subsemiring of $\mathbb{N}_0$, $A \cap \mathbb{N} \neq \emptyset$. So $A \neq \{0\}$. Conversely, assume that $0 \in A$ and $A \neq \{0\}$. Then $n \in A$ for some positive integer n. Let S be any subsemiring of $\mathbb{N}_0$. If $S = \{0\}$, then $A \cap S \neq \emptyset$. Assume that $S \neq \{0\}$. Then there exists a positive integer m such that $m \in S$. Clearly, $mn \in A \cap S$. Therefore, A is essential. $\square$

Remark 2.6. *Let $A = \{0, 2\} \cup \{4, 5, 6, \ldots\}$ and $B = \{0\} \cup \{3, 4, 5, \ldots\}$. It is clear that A, B and $A \cup B$ are subsemirings of $\mathbb{N}_0$. By Theorem 2.5, A, B and $A \cup B$ are essential. Note that $A \nsubseteq B$ and $B \nsubseteq A$.*

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