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On the 3-Distance Graph of Graphs

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Abstract

The 3-distance graph $D_3(G)$ of a graph G is the graph with vertex set $V(G)$ in which two vertices are adjacent if and only if the distance between them is 3 in the graph G. In this paper, we give some properties of the 3-distance graphs of graphs.

1 Introduction and Preliminaries

Let G be a graph. A subset D of $V(G)$ is a *dominating set* of G if for $v \in V(G) \setminus D, d_G(u, v) = 1$ for some $u \in D$. The minimum cardinality of all dominating sets is called the *domination number* of G, denoted by $\gamma(G)$. Later, 3-distance dominating sets were studied in [1]. A subset Q of $V(G)$ is

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a *3-distance dominating set* of G if for $v \in V(G) \setminus Q, d_G(u,v) = 3$ for some $u \in Q$. The *3-distance domination number* of G, denoted by $\gamma^3(G)$, is the minimum cardinality of all 3-distance dominating sets. The *3-distance graph* $D_3(G)$ of a graph G is the graph obtained from G by taking $V(D_3(G)) = V(G)$ and joining two vertices u, v in $D_3(G)$ if and only if they are at distance 3 in G ([2, 3]). So $d_G(u,v) = 3$ if and only if $d_{D_3(G)}(u,v) = 1$. In 2025, Musawi and Jafari [3] provided some conditions for the connectedness of $D_3(G)$. The purpose of this paper is to investigate some properties of the 3-distance graphs of graphs. We show some graphs G such that $D_3(G)$ and G are isomorphic.

2 Main results

Theorem 2.1. *Let G be a graph. The set of all 3-distance dominating sets of G and the set of all dominating sets of $D_3(G)$ coincide.*

Proof. Let Q be a 3-distance dominating set of G. Let $u \in V(D_3(G)) \setminus Q$. So $u \in V(G) \setminus Q$. Thus there exists $v \in Q$ such that $d_G(u,v) = 3$. Hence $d_{D_3(G)}(u,v) = 1$. Therefore Q is a dominating set of $D_3(G)$. Conversely, let D be a dominating set of $D_3(G)$. Let $u \in V(G) \setminus D$. Thus $u \in V(D_3(G)) \setminus D$. So there exists $v \in D$ such that $d_{D_3(G)}(u,v) = 1$. Hence $d_G(u,v) = 3$. Therefore D is a 3-distance dominating set of G. □

Corollary 2.2. *Let G be a graph. Then $\gamma(D_3(G)) = \gamma^3(G)$.*

Theorem 2.3. *Let G be a graph. Then diam$(G) \leq 2$ if and only if $D_3(G)$ is isolated.*

Proof. Assume that diam$(G) \leq 2$. So there are no $u, v \in V(G)$ such that $d_G(u,v) = 3$. Hence $E(D_3(G)) = \emptyset$. Therefore $D_3(G)$ is isolated. Conversely, assume $D_3(G)$ is isolated. Then there are no $u, v \in V(G)$ such that $d_{D_3(G)}(u,v) = 1$. So there are no $u, v \in V(G)$ such that $d_G(u,v) = 3$. Hence diam$(G) \leq 2$. □

Corollary 2.4. *Let G be a graph with diam$(G) \leq 2$. Then $D_3(G) = G$ if and only if G is isolated.*

Corollary 2.5. *The following statements hold.*

(1) If S_n is a star graph with n vertices, then $D_3(S_n)$ is isolated.
(2) If K_n is a complete graph with n vertices, then $D_3(K_n)$ is isolated.
(3) If W_n is a wheel graph with n vertices, then $D_3(W_n)$ is isolated.

(4) If F_n is a fan graph with n vertices, then $D_3(F_n)$ is isolated.

(5) If $G + H$ is the join of two graphs G and H, then $D_3(G + H)$ is isolated.

Theorem 2.6. *Let C_n be a cycle graph with n vertices. Then*

(1) If $n < 6$, then $D_3(C_n)$ is isolated.

(2) If $n = 6$, then $D_3(C_n)$ consists of 3 disjoint edges (a perfect matching).

(3) If $n > 6$ and $3 \mid n$, then $D_3(C_n)$ has 3 components, each of which is a cycle graph with $\frac{n}{3}$ vertices.

(4) If $n > 6$ and $3 \nmid n$, then $D_3(C_n)$ is a cycle graph with n vertices.

Proof. (1) If $n < 6$, then $\operatorname{diam}(C_n) \leq 2$. By Theorem 2.3, $D_3(C_n)$ is isolated.

(2) Assume that $n = 6$. Let $v_1, v_2, \ldots, v_6$ be the vertices of C_6 listed in cyclic order. Then $d(v_1, v_4) = d(v_2, v_5) = d(v_3, v_6) = 3$, and these are the only pairs of vertices at distance 3. Hence $D_3(C_6)$ consists of the 3 disjoint edges $\{v_1, v_4\}, \{v_2, v_5\}, \{v_3, v_6\}$, that is, a perfect matching.

(3) Assume that $n > 6$ and $3 \mid n$. It is easy to see that $D_3(C_n)$ has 3 components, each a cycle graph, namely $\{v_1, v_4, \ldots, v_{n-2}\}, \{v_2, v_5, \ldots, v_{n-1}\}$ and $\{v_3, v_6, \ldots, v_n\}$.

(4) Assume that $n > 6$ and $3 \nmid n$. Since $(3, n) = 1$, $\overline{3}$ is a generator of the group $(\mathbb{Z}_n, +)$. This means that $\{\overline{3}, \overline{6}, \ldots, \overline{3n}\} = \mathbb{Z}_n$. Then for each $k \in \{1, 2, \ldots, n\}$, there exists $t_k \in \{1, 2, \ldots, n\}$ such that $\overline{3k} = \overline{t_k}$. It is easy to see that $D_3(C_n)$ is a cycle graph $\{v_{t_1}, v_{t_2}, \ldots, v_{t_n}\}$. $\square$

Corollary 2.7. *Let G be a graph. If for any component C of G, C is isolated or C is a cycle graph with n vertices for some $n > 6$ and $3 \nmid n$, then $D_3(G)$ and G are isomorphic.*

Proof. This follows from Corollary 2.4 and Theorem 2.6 (4). $\square$

References

[1] J. A. Hassan, U. S. Iyah, N. M. Paradji, A. E. Gamorez, E. C. Ahmad, 3-distance domination numbers of some graphs, International Journal of Mathematics and Computer Science, **19**, no. 4, (2024), 981–986.

[2] O. Khormali, On the connectivity of k-distance graphs, Electronic Journal of Graph Theory and Applications, **5**, no. 1, (2017), 83–93.

[3] S. R. Musawi, S. H. Jafari, Connectivity of 3-distance graphs, Asian-European J. Math., **18**, no. 10, (2025), Article no. 2550038.