# Positive Solutions of Diophantine Equation 2/x+3/y+4/z = 1/3

SARANYA HANGSAWAT[1], RONNASON CHINRAM[2,*], PORAMIN PRABNARONG[2]
[1]Mathematics Program, Faculty of Science and Technology,
Songkhla Rajabhat University,
Songkhla 90000,
THAILAND

[2]Division of Computation Science, Faculty of Science,
Prince of Songkla University,
Hat Yai, Songkhla 90110,
THAILAND

*Corresponding Author

*Abstract:* - The Erdös-Straus conjecture states that for every positive integer $n \geq 2$, there exist three natural numbers $x$, $y$, and $z$, not necessarily distinct, so that $1/x + 1/y + 1/z = 4/n$. In this paper, we consider the Diophantine equation $2/x+3/y+4/z = 1/3$. We find positive solutions of this Diophantine equation.

*Key-Words:* - Diophantine equations, positive integer solutions, Erdös-Straus conjecture, Egyptian fractions, fractions, sums of fractions.

Received: May 25, 2025. Revised: July 19, 2025. Accepted: October 9, 2025. Published: December 31, 2025.

## 1 Introduction

Let $a,b,c,$ and $d$ be fixed positive integers. For a positive integer $n$, we consider the Diophantine equation:

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{d}{n}.$$

We consider $a = b = c = 1$ and $d = 4$, this Diophantine equation as the following form:

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{n}.$$

The Erdös-Straus conjecture states that for every positive integer $n \geq 2$, there exist three natural numbers $x$, $y$, and $z$, not necessarily distinct, so that $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{n}$. This conjecture is named after Paul Erdös and Ernst G. Straus, who formulated it in 1948, [1], [2]. This means that $4/n$ is a finite sum of distinct unit fractions; that is, the Erdös-Straus conjecture shows that $4/n$ is an Egyptian fraction.

For various positive integers $a,b,c$ and $d$, the methods of finding solutions of $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{d}{n}$ were presented, [3], [4], [5], [6], [7].

Moreover, the Diophantine equations have been a long-standing problem, and recently, several interesting problems have been solved, we can see some Diophantine equation in [8], [9], [10], [11].

In this paper, we will consider $a = 2$, $b = 3$, $c = 4$, and $d = 1$. Our purpose is to find positive solutions to the Diophantine equation $\frac{2}{x}+\frac{3}{y}+\frac{4}{z}=\frac{1}{3}$.

## 2 Problem Solutions

This section aims to find positive solutions to the Diophantine equation $\frac{2}{x}+\frac{3}{y}+\frac{4}{z}=\frac{1}{3}$. We get $\frac{2}{x}<\frac{1}{3}$, $\frac{3}{y}<\frac{1}{3}$, $\frac{4}{z}<\frac{1}{3}$. This implies that $x \geq 7, y \geq 10$, and $z \geq 13$.

We consider the following three cases.

**Case I.** $x \leq y \leq z$ or $x \leq z \leq y$

If $x \leq y \leq z$, then $\frac{2}{x}+\frac{3}{y}+\frac{4}{z} \leq \frac{9}{x}$.

If $x \leq z \leq y$, then $\frac{2}{x}+\frac{3}{y}+\frac{4}{z} \leq \frac{9}{x}$.

Therefore, $\frac{1}{3}=\frac{2}{x}+\frac{3}{y}+\frac{4}{z} \leq \frac{9}{x}$ which implies that $x \leq 27$. From $x \geq 7$, we get $7 \leq x \leq 27$. If $x = 7$, then $\frac{3}{y}+\frac{4}{z}=\frac{1}{21}$. Therefore, we have that

$$(y-63)(z-84)=5292.$$

We use the function $\tau$ to ensure that the total number of divisors of 5292 is 36. We have that the factors of 5292 are 1, 2, 3, 4, 6, 7, 9, 12, 14, 18, 21, 27, 28, 36, 42, 49, 54, 63, 84, 98, 108, 126, 147, 189, 196, 252, 294, 378, 441, 588, 756, 882, 1323, 1764, 2646 and 5292. Using factors of 5292, we have 36 cases as follows:

$$y-63=1, z-84=5292$$
$$y-63=2, z-84=2646$$
$$y-63=3, z-84=1764$$
$$y-63=4, z-84=1323$$
$$y-63=6, z-84=882$$
$$y-63=7, z-84=756$$
$$y-63=9, z-84=588$$
$$y-63=12, z-84=441$$
$$y-63=14, z-84=378$$
$$y-63=18, z-84=294$$
$$y-63=21, z-84=252$$
$$y-63=27, z-84=196$$
$$y-63=28, z-84=189$$
$$y-63=36, z-84=147$$
$$y-63=42, z-84=126$$
$$y-63=49, z-84=108$$
$$y-63=54, z-84=98$$
$$y-63=63, z-84=84$$
$$y-63=84, z-84=63$$
$$y-63=98, z-84=54$$
$$y-63=108, z-84=49$$
$$y-63=126, z-84=42$$
$$y-63=147, z-84=36$$
$$y-63=189, z-84=28$$
$$y-63=196, z-84=27$$
$$y-63=252, z-84=21$$
$$y-63=294, z-84=18$$
$$y-63=378, z-84=14$$
$$y-63=441, z-84=12$$
$$y-63=441, z-84=9$$
$$y-63=588, z-84=7$$
$$y-63=756, z-84=6$$
$$y-63=882, z-84=4$$
$$y-63=1323, z-84=3$$
$$y-63=2646, z-84=2$$
$$y-63=5292, z-84=1.$$

Then we have solutions $(x, y, z)$ in this case, as follows:
(7,64,5376), (7,65,2730), (7,66,1848), (7,67,1407) (7,69,966), (7,70,840), (7,72,672), (7,75,525), (7,77,462), (7,81,378), (7,84,336), (7,90,280), (7,91,273), (7,99,231), (7,105,210), (7,112,192), (7,117,182), (7,126,168), (7,147,147), (7,161,138), (7,171,133), (7,189,126), (7,210,120), (7,252,112), (7,259,111), (7,315,105), (7,357,102), (7,441,98), (7,504,96), (7,651,93), (7,819,91), (7,945,90), (7,1386,88), (7,1827,87), (7,2709,86) and (7,5355,85).

In the same manner,
if $x=8$, then $\frac{3}{y}+\frac{4}{z}=\frac{1}{12}$. We have:
$$(y-36)(z-48)=1728.$$

Since the factors of 1728 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54, 64, 72, 96, 108, 144, 192, 216, 288, 432, 576, 864, and 1728. Then we have solutions $(x, y, z)$ in this case as follows:
(8,37,1776), (8,38,912), (8,39,624), (8,40,480), (8,42,336), (8,44,264), (8,45,240), (8,48,192), (8,52,156), (8,54,144), (8,60,120), (8,63,112), (8,68,102), (8,72,96), (8,84,84), (8,90,80), (8,100,75), (8,108,72), (8,132,66), (8,144,64), (8,180,60), (8,228,57), (8,252,56), (8,324,54), (8,468,52), (8,612,51), (8,900,50) and (8,1764,49).

If $x=9$, then $\frac{3}{y}+\frac{4}{z}=\frac{1}{9}$. We have
$$(y-27)(z-36)=972.$$

The factors of 972 are 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 81, 108, 162, 243, 324, 486, 972. The solutions $(x, y, z)$ in this case, are:
(9,28,1008), (9,29,522), (9,30,360), (9,31,279), (9,33,198), (9,36,144), (9,39,117), (9,45,90), (9,54,72), (9,63,63), (9,81,54), (9,108,48), (9,135,45), (9,189,42), (9,270,40), (9,351,39), (9,513,38) and (9,999,37).

If $x=10$, then $\frac{3}{y}+\frac{4}{z}=\frac{2}{15}$. We have:
$$(2y-45)(z-30)=1350.$$

The factors of 1350 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 25, 27, 30, 45, 50, 54, 75, 90, 135, 150, 225, 270, 450, 675, and 1350. However, in this case, z must be an even number. In this case, we have that the number of solutions $(x, y, z)$ is 12. We have the solutions as follows:
(10,23,1380), (10,24,480), (10,25,300), (10,27,180), (10,30,120), (10,35,84), (10,36,80), (10,45,60), (10,60,48), (10,90,40), (10,135,36) and (10,360,32).

If $x=12$, then $\frac{3}{y}+\frac{4}{z}=\frac{1}{6}$. We have:
$$(y-18)(z-24)=432.$$

The solutions $(x, y, z)$ are:
(12,19,456), (12,20,240), (12,21,168), (12,22,132), (12,24,96), (12,26,78), (12,27,72), (12,30,60), (12,34,51), (12,36,48), (12,42,42), (12,45,40), (12,54,36), (12,66,33), (12,72,32), (12,90,30), (12,126,28), (12,162,27), (12,234,26) and (12,450,25).

If $x=15$, then $\frac{3}{y}+\frac{4}{z}=\frac{1}{5}$. We have:
$$(y-15)(z-20)=300.$$

Since $x \le y \le z$ or $x \le z \le y$, the solutions $(x, y, z)$ are
(15,16,320),(15,17,170),(15,18,120),(15,19,95),
(15,20,80),(15,21,70),(15,25,50),(15,27,45),
(15,30,40),(15,35,35),(15,40,32),(15,45,30),
(15,65,26),(15,75,25),(15,90,24),(15,115,23),
(15,165,22) and (15,315,21).

If $x = 18$, then $\frac{3}{y} + \frac{4}{z} = \frac{2}{9}$. We have:

$$(2y - 27)(z - 18) = 486.$$

Since $x \le y \le z$ or $\le z \le y$ ,

The solutions $(x, y, z)$ are:
(18,14,504), (18,15,180), (18,18,72), (18,27,36),
(18,54,24) and (18,135,20).

If $x = 11$, then $\frac{3}{y} + \frac{4}{z} = \frac{5}{33}$.

**Case $z \le y$.** We get $\frac{5}{33} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{z}$, so $11 \le z \le 46$. Since $y$ is a positive integer, the solutions $(x, y, z)$ are (11,891,27), (11,165,30) and (11,99,33).

**Case $y \le z$.** We get $\frac{5}{33} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{y}$, so $11 \le y \le 46$.

Since $z$ is a positive integer, the solutions $(x, y, z)$ are (11,20,2640), (11,21,462), (11,22,264),
(11,27,99), (11,33,66) and (11,44,48).

If $x = 13$, then $\frac{3}{y} + \frac{4}{z} = \frac{7}{39}$.

**Case $z \le y$.** We get $\frac{7}{39} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{z}$, so $13 \le z \le 39$. Since $y$ is a positive integer, the solutions $(x, y, z)$ are (13,234,24),(13,117,26),(13,65,30) and (13,39,39).

**Case $y \le z$.** We get $\frac{7}{39} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{y}$, so $13 \le y \le 46$. Since $z$ is a positive integer, the solutions $(x, y, z)$ are (13,17,1326), (13,18,312) and (13,39,39).

If $x = 14$, then $\frac{3}{y} + \frac{4}{z} = \frac{4}{21}$.

**Case $z \le y$.** We get $\frac{4}{21} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{z}$, so $14 \le z \le 37$. Since $y$ is a positive integer, the solutions $(x, y, z)$ are (14,126,24) and (14,63,28).

**Case $y \le z$.** We get $\frac{4}{21} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{y}$, so $14 \le y \le 37$. Since $z$ is a positive integer, the solutions $(x, y, z)$ are (14,16,1344),(14,18,168), (14,21,84) and (14,28,48).

If $x = 16$, then $\frac{3}{y} + \frac{4}{z} = \frac{5}{24}$.

**Case $z \le y$.** We get $\frac{5}{24} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{z}$, so $16 \le z \le 37$. Since $y$ is a positive integer, the solutions $(x, y, z)$ are (16,360,20), (16,168,21), (16,72,24), (16,40,30) and (16,36,32).

**Case $y \le z$.** We get $\frac{5}{24} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{y}$, so $16 \le y \le 37$. Since $z$ is a positive integer, the solutions $(x, y, z)$ are (16,16,192),(16,18,96) and (16,24,48).

If $x = 17$, then $\frac{3}{y} + \frac{4}{z} = \frac{11}{51}$.

**Case $z \le y$.** We get $\frac{11}{51} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{z}$, so $17 \le z \le 32$.
Since $y$ is a positive integer, the solutions $(x, y, z)$ is (17,119,21).

**Case $y \le z$.** We get $\frac{11}{51} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{y}$, so $17 \le y \le 32$. Since $z$ is a positive integer, the solutions $(x, y, z)$ is (17,17,102).

If $x = 19$, then $\frac{3}{y} + \frac{4}{z} = \frac{13}{57}$.

**Case $z \le y$.** We get $\frac{13}{57} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{z}$, so $19 \le z \le 31$. Since $y$ is a positive integer, the solutions $(x, y, z)$ is (19,171,19).

**Case $y \le z$.** We get $\frac{13}{57} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{y}$, so $19 \le y \le 31$. Since $z$ is a positive integer, the solutions $(x, y, z)$ is (19,19,57).

If $x = 20$, then $\frac{3}{y} + \frac{4}{z} = \frac{7}{30}$.

**Case $z \le y$.** We get $\frac{7}{30} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{z}$, so $20 \le z \le 30$.
Since $y$ is a positive integer, the solutions $(x, y, z)$ are $(20, 90, 20), (20, 70, 21)$, (20,45,24) and (20,30,30).

**Case $y \le z$.** We get $\frac{7}{30} = \frac{3}{y} + \frac{4}{z} \le \frac{7}{y}$, so $20 \le y \le 30$.
Since $z$ is a positive integer, the solutions $(x, y, z)$ are (20,20,48) and (20,30,30).

**Case II.** $y < x \le z$ or $y \le z < x$

If $y < x \le z$, then $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} \le \frac{9}{y}$.

If $y \le z < x$, then $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} \le \frac{9}{y}$.

Therefore, $\frac{1}{3} = \frac{2}{x} + \frac{3}{y} + \frac{4}{z} \le \frac{9}{y}$ which implies that $y < 27$. From $y \ge 10$, we get $10 \le y \le 26$. If $y = 10$, then $\frac{2}{x} + \frac{4}{z} = \frac{1}{30}$. We have:

$$(x - 60)(z - 120) = 7200.$$

We use the function $\tau$ to ensure that the total number of divisors of 7200 is 54. The factors of 7200 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 32, 36, 40, 45, 48, 50, 60, 72, 75, 80, 90, 96, 100, 120, 144, 150, 160, 180, 200, 225, 240, 288, 300, 360, 400, 450, 480, 600, 720, 800, 900, 1200, 1440, 1800, 2400, 3600, 7200. Using factors of 7200, we have 54 solutions $(x, y, z)$ as follows:
(61,10,7320), (62,10,3720), (63,10,2520), (64,10,1920), (65,10,1560), (66,10,1320), (68,10,1020), (69,10,920), (70,10,840), (72,10,720), (75,10,600), (76,10,570), (78,10,520), (80,10,480), (84,10,420), (85,10,408), (90,10,360), (92,10,345), (96,10,320), (100,10,300), (105,10,280), (108,10,270), (110,10,264), (120,10,240), (132,10,220), (135,10,216), (140,10,210), (150,10,200), (156,10,195), (160,10,192), (180,10,180), (204,10,170), (210,10,168), (220,10,165), (240,10,160), (260,10,156), (285,10,152), (300,10,150), (348,10,145), (360,10,144), (420,10,140), (460,10,138), (510,10,136), (540,10,135), (660,10,132), (780,10,130), (860,10,129), (960,10,128), (1260,10,126), (1500,10,125), (1860,10,124), (2460,10,123), (3660,10,122) and (7260,10,121).

If $y = 11$, then $\frac{1}{x} + \frac{2}{z} = \frac{1}{33}$. We have:

$$(x - 33)(z - 66) = 2178.$$

The solutions $(x, y, z)$ are:
(34,11,2244), (35,11,1155), (36,11,792), (39,11,429), (42,11,308), (44,11,264), (51,11,187), (55,11,165), (66,11,132), (99,11,99), (132,11,88), (154,11,84), (231,11,77), (275,11,75), (396,11,72), (759,11,69), (1122,11,68) and (2211,11,67).

If $y = 12$, then $\frac{2}{x} + \frac{4}{z} = \frac{1}{12}$. We have:

$$(x - 24)(z - 48) = 1152.$$

The solutions $(x, y, z)$ are:
(25,12,1200), (26,12,624), (27,12,432), (28,12,336), (30,12,240), (32,12,192), (33,12,176), (36,12,144), (40,12,120), (42,12,112), (48,12,96), (56,12,84), (60,12,80), (72,12,72), (88,12,66), (96,12,64), (120,12,60), (152,12,57), (168,12,56), (216,12,54), (312,12,52), (408,12,51), (600,12,50) and (1176,12,49).

If $y = 13$, then $\frac{1}{x} + \frac{2}{z} = \frac{2}{39}$. We have:

$$(2x - 39)(z - 39) = 1521.$$

The solutions $(x, y, z)$ are:
(20,13,1560),(21,13,546),(24,13,208), (26,13,156), (39,13,78),(78,13,52),(104,13,48),(273,13,42) and (780,13,40).

If $y = 15$, then $\frac{1}{x} + \frac{2}{z} = \frac{1}{15}$. We have:

$$(x - 15)(z - 30) = 450.$$

The solutions $(x, y, z)$ are:
(16,15,480), (17,15,255), (18,15,180), (20,15,120), (21,15,105), (24,15,80), (25,15,75), (30,15,60), (33,15,55), (40,15,48), (45,15,45), (60,15,40), (65,15,39), (90,15,36), (105,15,35), (165,15,33), (240,15,32) and (465,15,31).

If $y = 18$, then $\frac{2}{x} + \frac{4}{z} = \frac{1}{6}$. We have:

$$(x - 12)(z - 24) = 288.$$

The solutions $(x, y, z)$ are:
(13,18,312), (14,18,168), (15,18,120), (16,18,96), (18,18,72), (20,18,60), (21,18,56), (24,18,48), (28,18,42), (30,18,40), (36,18,36), (44,18,33), (48,18,32), (60,18,30), (84,18,28), (108,18,27), (156,18,26) and (300,18,25).

If $y = 14$, then $\frac{2}{x} + \frac{4}{z} = \frac{5}{42}$.

**Case $x \le z$.** We get $\frac{5}{42} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{x}$, so $15 \le x \le 50$.
Since $z$ is a positive integer, the solutions $(x, y, z)$ are:
(17,14,2856), (18,14,504), (20,14,210), (21,14,168), (24,14,112), (28,14,84), (36,14,63) and (42,14,56).

**Case $z \le x$.** We get $\frac{5}{42} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{z}$, so $14 \le z \le 50$.
Since $x$ is a positive integer, the solutions $(x, y, z)$ are:
(1428,14,34), (420,14,35), (252,14,36), (105,14,40), (84,14,42) and (56,14,48).

If $y = 16$, then $\frac{2}{x} + \frac{4}{z} = \frac{7}{48}$.

**Case $x \le z$.** We get $\frac{7}{48} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{x}$, so $17 \le x \le 41$.
Since $z$ is a positive integer, the solutions $(x, y, z)$ are (24,16,64) and (32,16,48).

**Case $z \le x$.** We get $\frac{7}{48} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{z}$, so $16 \le z \le 41$.

Since $x$ is a positive integer, the solutions $(x, y, z)$ are (672,16,28),(160,16,30) and (96,16,32).

If $y = 17$, then $\frac{2}{x} + \frac{4}{z} = \frac{8}{51}$.

**Case $x \le z$.** We get $\frac{8}{51} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{x}$, so $18 \le x \le 38$.
Since $z$ is a positive integer, there is no solution.

**Case $z \le x$.** We get $\frac{8}{51} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{z}$, so $17 \le z \le 38$.
Since $x$ is a positive integer, the solutions $(x, y, z)$ are (663,17,26), (85,17,30) and (51,16,34).
If $y = 19$, then $\frac{2}{x} + \frac{4}{z} = \frac{10}{57}$.

**Case $x \le z$.** We get $\frac{10}{57} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{x}$, so $20 \le x \le 34$.
Since $z$ is a positive integer, there is no solution.

**Case $z \le x$.** We get $\frac{10}{57} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{z}$, so $19 \le z \le 34$.
Since $x$ is a positive integer, the solutions $(x, y, z)$ are (1311,19,23) and (228,19,24).

If $y = 20$, then $\frac{2}{x} + \frac{4}{z} = \frac{11}{60}$.

**Case $x \le z$.** We get $\frac{11}{60} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{x}$, so $21 \le x \le 33$.
Since $z$ is a positive integer, the solutions $(x, y, z)$ is (24,20,40).

**Case $z \le x$.** We get $\frac{11}{60} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{z}$, so $20 \le z \le 33$.
Since $x$ is a positive integer, the solutions $(x, y, z)$ are (1320,20,22),(120,20,24) and (40,20,30).

If $y = 21$, then $\frac{2}{x} + \frac{4}{z} = \frac{4}{21}$.

**Case $x \le z$.** We get $\frac{4}{21} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{x}$, so $22 \le x \le 32$.
Since $z$ is a positive integer, there is no solution.

**Case $z \le x$.** We get $\frac{4}{21} = \frac{2}{x} + \frac{4}{z} \le \frac{6}{z}$, so $21 \le z \le 32$.
Since $x$ is a positive integer, the solutions $(x, y, z)$ are (231,21,22), (84,20,24), (42,20,28) and (35,20,30).

**Case III.** $z < x \le y$ or $z \le y < x$

If $z < x \le y$, then $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} \le \frac{9}{z}$.
If $z \le y < x$, then $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} \le \frac{9}{z}$.

Therefore, $\frac{1}{3} = \frac{2}{x} + \frac{3}{y} + \frac{4}{z} \le \frac{9}{z}$ which implies that $z < 27$. From $z \ge 13$, we get $13 \le z \le 26$. If $z = 13$, then $\frac{2}{x} + \frac{3}{y} = \frac{1}{39}$. We have:

$$(x - 78)(y - 117) = 9126.$$

We use the function $\tau$ to ensure that the total number of divisors of 9126 is 24. Using factors of 9126, we have the solutions $(x, y, z)$ are:
(79,9243,13), (80,4680,13), (81,3159,13), (84,1638,13), (87,1131,13), (91,819,13), (96,624,13), (104,468,13), (105,455,13), (117,351,13), (132,286,13), (156,234,13), (195,195,13), (247,171,13), (312,156,13), (416,144,13), (429,143,13), (585,136,13), (780,130,13), (1092,126,13), (1599,123,13), (3120,120,13), (4641,119,13) and (9204,118,13).

If $z = 14$, then $\frac{2}{x} + \frac{3}{y} = \frac{1}{21}$. We have:

$$(x - 42)(y - 63) = 2646.$$

The solutions $(x, y, z)$ are:
(43,2709,14), (44,1386,14), (45,945,14), (48,504,14), (49,441,14), (51,357,14), (56,252,14), (60,210,14), (63,189,14), (69,161,14), (84,126,14), (91,117,14), (96,112,14), (105,105,14), (140,90,14), (168,84,14), (189,81,14), (231,77,14), (336,72,14), (420,70,14), (483,69,14), (924,66,14), (1365,65,14) and (2688,64,14).

If $z = 15$, then $\frac{2}{x} + \frac{3}{y} = \frac{1}{15}$. We have:

$$(x - 30)(y - 45) = 1350.$$

The solutions $(x, y, z)$ are:
(31,1395,15), (32,720,15), (33,495,15), (35,315,15), (36,270,15), (39,195,15), (40,180,15), (45,135,15), (48,120,15), (55,99,15), (57,95,15), (60,90,15), (75,75,15), (80,72,15), (84,70,15), (105,63,15), (120,60,15), (165,55,15), (180,54,15), (255,51,15), (300,50,15), (480,48,15), (705,47,15) and (1380,46,15).

If $z = 16$, then $\frac{2}{x} + \frac{3}{y} = \frac{1}{12}$. We have:

$$(x - 24)(y - 36) = 864.$$

The solutions $(x, y, z)$ are:
(25,900,16), (26,468,16), (27,324,16), (28,252,16), (30,180,16), (32,144,16), (33,132,16), (36,108,16), (40,90,16), (42,84,16), (48,72,16), (51,68,16), (56,63,16), (60,60,16), (72,54,16), (78,52,16), (96,48,16), (120,45,16), (132,44,16), (168,42,16), (240,40,16), (312,39,16), (456,38,16) and (888,37,16).

If $z = 18$, then $\frac{2}{x} + \frac{3}{y} = \frac{1}{9}$. We have:

$$(x - 18)(y - 27) = 486.$$

The solutions $(x, y, z)$ are:
(19,513,18), (20,270,18), (21,189,18), (24,108,18), (27,81,18), (36,54,18), (45,45,18), (72,36,18), (99,33,18), (180,30,18), (261,29,18) and (504,28,18).

If $z = 20$, then $\frac{2}{x} + \frac{3}{y} = \frac{2}{15}$. We have:

$$(x - 15)(2y - 45) = 675.$$

The solutions $(x, y, z)$ are:
(16,360,20), (18,135,20), (20,90,20), (24,60,20), (30,45,20), (40,36,20), (42,35,20), (60,30,20), (90,27,20), (150,25,20), (240,24,20) and (690,23,20).

If $z = 17$, then $\frac{2}{x} + \frac{3}{y} = \frac{5}{51}$.

**Case $x \le y$.** We get $\frac{5}{51} = \frac{2}{x} + \frac{3}{y} \le \frac{5}{x}$, so $18 \le x \le 51$.
Since $y$ is a positive integer, the solutions $(x, y, z)$ are (21,1071,17), (24,204,17) and (51,51,17).

**Case $y \le x$.** We get $\frac{5}{51} = \frac{2}{x} + \frac{3}{y} \le \frac{5}{y}$, so $18 \le y \le 51$.
Since $x$ is a positive integer, the solutions $(x, y, z)$ are (1581,31,17), (204,34,17) and (136,36,17).

If $z = 19$, then $\frac{2}{x} + \frac{3}{y} = \frac{7}{57}$.

**Case $x \le y$.** We get $\frac{7}{57} = \frac{2}{x} + \frac{3}{y} \le \frac{5}{x}$, so $19 \le x \le 41$.
Since $y$ is a positive integer, the solutions $(x, y, z)$ are (19,171,19) and (24,76,19).

**Case $y \le x$.** We get $\frac{7}{57} = \frac{2}{x} + \frac{3}{y} \le \frac{5}{y}$, so $20 \le y \le 41$.
Since $x$ is a positive integer, the solutions $(x, y, z)$ is (171,27,19).

If $z = 21$, then $\frac{2}{x} + \frac{3}{y} = \frac{1}{7}$.

**Case $x \le y$.** We get $\frac{1}{7} = \frac{2}{x} + \frac{3}{y} \le \frac{5}{x}$, so $21 \le x \le 35$.
Since $y$ is a positive integer, the solutions $(x, y, z)$ are (21,63,21), (28,42,21) and (35,35,21).

**Case $y \le x$.** We get $\frac{1}{7} = \frac{2}{x} + \frac{3}{y} \le \frac{5}{y}$, so $22 \le y \le 35$.
Since $x$ is a positive integer, the solutions $(x, y, z)$ are (308,22,21), (161,23,21), (112,24,21), (63,27,21), (56,28,21) and (35,35,21).

On the other hand, we can compute directly that $(x, y, z)$ satisfy the equation $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} = \frac{1}{3}$. Hence, we obtain solutions of the Diophantine equation $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} = \frac{1}{3}$ where $x, y,$ and $z$ are positive integers.

Finally, we short notes as follows:
If $(x_0, y_0, z_0)$ is a positive integral solution of the Diophantine equation $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} = \frac{1}{3}$ such that $2|z_0$, then $\left(\frac{z_0}{2}, y_0, 2x_0\right)$ is also a positive integral solution of the equation $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} = \frac{1}{3}$.

For example, if (690,23,20) is a solution, then (10,23,1380) is also a solution.

## 3 Conclusion

In this paper, we find the positive solutions of the Diophantine equation $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} = \frac{1}{3}$.

*References:*

[1] E. J. Ionascu and A. Wilson, On the Erdös-Straus Conjecture, *Revue Roumaine de Mathematiques Pures et Appliquees*, Vol. 56, 2011, pp. 21-30.

[2] S. Subburam and A.Togbé, A note on the Erdös-Straus conjecture, *Periodica Mathematica Hungarica*, Vol. 72, No. 1, 2016, pp. 43-49.

[3] J. F. T. Rabago and R. P. Tagle, On the Areas and Volume of a Certain Regular Solid and

the Diophantine Equation 1/2 = 1/x + 1/y + 1/z, *Notes on Number Theory and Discrete Mathematics*, Vol. 19, 2013, pp. 28-32.

[4] J. Sandor, A Note on a Diophantine Equation, *Notes on Number Theory and Discrete Mathematics*, Vol. 19, 2013, pp. 1-3.

[5] K. Srimud, N. Makate, T. Ampawa and T. Jantree, On the Diophantine equation 2/x + 3/y + 4/z = 1/2, *Progress in Applied Science and Technology*, Vol. 12, No. 1, 2022, pp. 11-16.

[6] R. Chinram, K. Sirikantisophon, S. Kaewchay, Positive integers of the Diophantine equation 1/x+2/y+3/z=1/3, *International Journal of Mathematics and Computer Science*, Vol. 17, No.3, 2022, pp. 1051-1059.

[7] S. Tadee and S. Poopra, On the Diophantine 1/x+2/y+3/z=1/3, *International Journal of Mathematics and Computer Science*, Vol. 18, No.2, 2023, pp. 173-177.

[8] T. Buatong, R. Chinram, On Diophantine equation $n^x+(9p)^y=z^2$, *International Journal of Mathematics and Computer Science,* Vol. 20, No. 3, 2025, pp. 897-900.

[9] S. Tadee, On the Diophantine Equation $n^x + 10^y = z^2$, *Wseas Transactions on Mathematics*, Vol. 22, 2023, pp. 150–153.

[10] S. Luengaksorn, R. Chinram, Solutions of the Diophantine equation $n^x + (7p)^y = z^2$, *Journal of Interdisciplinary Mathematics, Open source preview*, Vol. 28, No.5, 2025, pp. 2005–2010.

[11] W. Orosram, C. Comemuang, On the diophantine equation $8^x + n^y = z^2$, *Wseas Transactions on Mathematics*, Vol.19, 2020, pp. 520–522.

**Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)**

The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

**Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself**

No funding was received for conducting this study.

**Conflict of Interest**

The authors have no conflicts of interest to declare that are relevant to the content of this article.