EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 3, Article Number 6259
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Almost n-ary Subsemigroups and Fuzzy Almost n-ary Subsemigroups of n-ary Semigroups

Ronnason Chinram[1], Pattarawan Singavananda[2,*]

[1] *Division of Computational Science, Faculty of Science, Prince of Songkla University, Hat Yai, Songkhla 90110, Thailand*
[2] *Mathematics Program, Faculty of Science and Technology, Songkhla Rajabhat University, Songkhla 90000, Thailand*

Abstract. An n-ary semigroup is a non-empty set with an associative n-ary operation. Semigroups and ternary semigroups are special cases of n-ary semigroups where $n = 2$ and $n = 3$, respectively. In this study, we introduce and explore the notions of almost n-ary subsemigroups and their fuzzy counterparts, termed fuzzy almost n-ary subsemigroups, within the framework of n-ary semigroups. Moreover, we demonstrate certain relationships between almost n-ary subsemigroups and fuzzy almost n-ary subsemigroups.

2020 Mathematics Subject Classifications: 20N15, 03E72

Key Words and Phrases: n-ary semigroups, almost n-ary subsemigroups, fuzzy almost n-ary subsemigroups, minimal, prime, semiprime.

1. Introduction

A fuzzy subset, also known as a fuzzy set, is a generalization of the classical set. A fuzzy set is represented by a membership function of all elements in a universal set assigning values in the closed interval $[0, 1]$. The concept of fuzzy sets was first introduced by Zadeh [1] in 1965. Zadeh's pioneering ideas have found widespread applications across various fields, including mathematics, computer science, and engineering. The concept of fuzzy sets has been extensively applied in the study of various algebraic structures.

The generalization of binary algebraic structures to n-ary structures was first initiated by Kasner [2] in 1904. In this paper, we focus on n-ary semigroups. Notably, semigroups and ternary semigroups arise as special cases of n-ary semigroups for $n = 2$ and $n = 3$, respectively. The notion of n-ary semigroups has its origins in the investigation of algebraic structures that extend the classical frameworks of semigroups and ternary semigroups. However, an n-ary semigroup does not necessarily reduce to a semigroup or a ternary

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v18i3.6259

Email addresses: ronnason.c@psu.ac.th (R. Chinram), pattarawan.pe@skru.ac.th (P. Singavananda)

semigroup. For $n \geq 3$, Dudek [3] also investigated the properties of ideals and some elements of n-ary semigroups containing an idempotent. In 2018, ideals of fuzzy points n-ary semigroups were studied by Solano et al. [4]. In the next year, Couceiro and Devillet [5] showed that every quasitrivial n-ary semigroup is reducible to a binary semigroup, and provided necessary and sufficient conditions for such a reduction to be unique. In 2020, Somsup and Leerawat [6] further investigated congruences and homomorphisms on n-ary semigroups. Later, Pornsurat and Pibaljommee [7] introduced the notions of a right regularity, a right weak regularity and a complete regularity of n-ary semigroups and characterized these regularities. In the same year, some reducibility of n-ary semigroups were investigated in [8]. In 2023, Daengsaen and Leeratanavalee [9] demonstrated that every n-ary semigroup that is both regular and intra-regular can be decomposed into a semilattice of i-simple and regular n-ary semigroups, and the reverse assertion also holds.

Almost ideals on semigroups were first studied by Grosek and Satko [10] in 1980. Later, Wattanatripop et al. [11] introduced almost fuzzy ideals of semigroups and investigated relationships between almost ideals and almost fuzzy ideals. Additionally, Khamrot and Gaketem [12] and [13] introduced the concepts of bipolar fuzzy almost ideals and picture fuzzy almost ideals in semigroups, respectively. Subsequently, the concepts of almost and fuzzy almost ideals were applied to several subalgebras within various algebraic structures. For instance, almost subsemigroups and fuzzy almost subsemigroups in semigroups [14]; almost subsemigroups and fuzzy almost ternary subsemigroups in ternary semigroups [15]; almost subsemirings and fuzzy almost subsemirings in semirings [16]; and almost ternary subsemirings and fuzzy almost ternary subsemirings in ternary semirings [17] etc.

This paper aims to generalize the findings presented in [14] and [15]. Basic notations and definitions are provided in Section 2. In section 3, we extend the main results. We introduce the concepts of almost n-ary subsemigroups and fuzzy almost n-ary subsemigroups of n-ary semigroups, and present their properties. Moreover, we establish some relationships between almost n-ary subsemigroups and fuzzy almost n-ary subsemigroups.

2. Preliminaries

The aim of this section is to review some notations and definitions of n-ary semigroups and fuzzy sets.

2.1. n-ary semigroups

To ensure completeness, we state some definitions in the same fashion as found in [3] and [18] which are used throughout this paper. First, we recall the definition of an n-ary semigroup, where n is a positive integer such that $n \geq 2$.

A nonempty set A together with an n-ary operation given by $f : A^n \to A$, where $n \geq 2$, is called an *n-ary groupoid* and is denoted by the notation (A, f). According to the general convention used in the symbols of n-ary groupoids, the sequence of elements $a_i, a_{i+1}, \ldots, a_j$ is denoted by a_i^j. In the case $j < i$, it is the empty symbol. If $a_{i+1} = a_{i+2} = \cdots = a_{i+t} = a$, then we will write a^t instead of a_{i+1}^{i+t}. In this convention, we have

$$f(a_1, a_2, \ldots, a_n) = f(a_1^n)$$

and

$$f(a_1, \ldots, a_i, \underbrace{a \ldots, a}_{t}, a_{i+t+1}, \ldots, a_n) = f(a_1^i, a^t, a_{i+t+1}^n).$$

An n-ary groupoid (A, f) is called (i, j)-*associative* if

$$f(a_1^{i-1}, f(a_i^{n+i-1}), a_{n+i}^{2n-1}) = f(a_1^{j-1}, f(a_j^{n+j-1}), a_{n+j}^{2n-1})$$

holds for all $a_1, a_2, \ldots, a_{2n-1} \in A$. The n-ary operation f is called *associative* if the above identity holds for every $1 \leq i \leq j \leq n$. In the case of the n-ary operation f is associate, (A, f) is called an *n-ary semigroup.*

A nonempty subset S of an n-ary semigroup (A, f) is called an *n-ary subsemigroup* of A if $f(a_1^n) \in S$ for all $a_1, a_2, \ldots, a_n \in S$. For nonempty subsets $S_1, S_2, \ldots, S_n$ of A, let

$$f(S_1^n) := \{f(a_1^n) \mid a_i \in S_i \text{ for all } i \in \{1, 2, \ldots, n\}\}.$$

If $S_1 = \{a_1\}$, then we write $f(\{a_1\}, S_2^n)$ as $f(a_1, S_2^n)$, and similarly in another case such as we write $f(\{a_1\}, S_2^{n-1}, \{a_n\})$ as $f(a_1, S_2^{n-1}, a_n)$ and so on. For any subset S of A, we let

$$f(S^n) = \{f(a_1^n) \mid a_1, a_2, \ldots, a_n \in S\}$$

and we let

$$f(A^n) = \{f(a_1^n) \mid a_1, a_2, \ldots, a_n \in A\}.$$

2.2. Fuzzy subsets

A fuzzy subset of a set A is defined as a membership function from A into the closed unit interval $[0, 1]$. We now recall some notations in fuzzy sets, as presented in [19]. Let g and h be two fuzzy subsets of a nonempty set A.

1. The intersection of g and h, denoted by $g \cap h$, is a fuzzy subset of A defined by $(g \cap h)(a) = \min\{g(a), h(a)\}$ for all $a \in A$.

2. The union of g and h, denoted by $g \cup h$, is a fuzzy subset of A defined by $(g \cup h)(a) = \max\{g(a), h(a)\}$ for all $a \in A$.

3. $g \subseteq h$ if $g(a) \leq h(a)$ for all $a \in A$.

For a fuzzy subset g of A, the support of g is defined by

$$supp(g) = \{a \in A \mid g(a) \neq 0\}.$$

The *characteristic mapping* of a subset S of A is a fuzzy subset of A defined by

$$\chi_S(a) = \begin{cases} 1 & a \in S, \\ 0 & a \notin S. \end{cases}$$

A fuzzy subset g of an n-ary semigroup A is called a *fuzzy n-ary subsemigroup* of A if $g(a_1^n) \geq \min\{g(a_1), g(a_2), \ldots, g(a_n)\}$ for all $a_1, a_2, \ldots, a_n \in A$.

Let $\mathcal{F}(A)$ be the set of all fuzzy subsets in an n-ary semigroup A. Define n-ary operator $\mathfrak{f}$ on $\mathcal{F}(A)$ by

$$\mathfrak{f}(g_1^n)(a) := \mathfrak{f}(g_1, g_2, \ldots, g_n)(a) = \begin{cases} \sup\limits_{a=f(a_1^n)} \min\{g_1(a_1), g_2(a_2), \ldots, g_n(a_n)\} & \text{if } a \in f(A^n), \\ 0 & \text{otherwise,} \end{cases}$$

for all $g_1, g_2, \ldots, g_n \in \mathcal{F}(A)$ and $a \in A$.

Proposition 1. *A fuzzy subset g of an n-ary semigroup A is a fuzzy n-ary subsemigroup of A if and only if $\mathfrak{f}(g^n) \subseteq g$.*

3. Main Results

3.1. Almost n-ary subsemigroups

We begin by introducing the definition of almost n-ary subsemigroups of n-ary semigroups.

Definition 1. A nonempty subset S of an n-ary semigroup A is called an *almost n-ary subsemigroup* of A if $f(S^n) \cap S \neq \emptyset$.

Every n-ary subsemigroup of an n-ary semigroup A is clearly an almost n-ary subsemigroup of A.

Example 1. We consider an n-ary semigroup $\mathbb{N}$ under the usual n-ary multiplication of integers. Let $S = \{2, 2^n\}$ and $T = \{2^n, 2^{n^2}\}$. Clearly, S and T are almost n-ary subsemigroups but are not n-ary subsemigroups of $\mathbb{N}$. However, $S \cap T = \{2^n\}$ is not an almost n-ary subsemigroup of $\mathbb{N}$.

From Example 1, we can draw the following conclusions.

(1) In general, an almost n-ary subsemigroup of an n-ary semigroup A need not be a n-ary subsemigroup of A.

(2) The intersection of almost n-ary subsemigroups of an n-ary semigroup A need not be an almost n-ary subsemigroup of A.

Theorem 1. *Let S be an almost n-ary subsemigroup of an n-ary semigroup A. If T be a nonempty subset of A such that $S \subseteq T$, then T is also an almost n-ary subsemigroup of A.*

Proof. Let S be an almost n-ary subsemigroup of A and T be a nonempty subset of A such that $S \subseteq T$. Thus $f(S^n) \cap S \neq \emptyset$. Since $S \subseteq T$, $f(S^n) \cap S \subseteq f(T^n) \cap T$. This implies that $f(T^n) \cap T \neq \emptyset$. It conclude that T is an almost n-ary subsemigroup of A.

The following corollary directly follows from Theorem 1.

Corollary 1. *The union of almost n-ary subsemigroups of an n-ary semigroup A is also an almost n-ary subsemigroup of A.*

An element a of an n-ary semigroup A is called a *selfpotent* if $a = f(a^n)$.

Proposition 2. *Let a be any element of a n-ary semigroup A.*

(1) If a is a selfpotent, then $\{a\}$ is an almost n-ary subsemigroup of A.

(2) If a is not a selfpotent, then $\{a, f(a^n)\}$ is an almost n-ary subsemigroup of A.

3.2. Fuzzy almost n-ary subsemigroups

In this subsection, we define fuzzy almost n-ary subsemigroups of n-ary semigroups and present their notable properties. A fuzzy subset g of an n-ary semigroup A is called a *zero fuzzy subset* if $g(a) = 0$ for all $a \in A$. If there exists $a \in A$ such that $g(a) \neq 0$, then g is called a *nonzero fuzzy subset* of A and we use the notation $g \neq 0$.

Definition 2. A fuzzy subset g of an n-ary semigroup A is called a *fuzzy almost n-ary subsemigroup* of A if $\mathfrak{f}(g^n) \cap g$ is not a zero fuzzy subset of A.

It is evident that a zero fuzzy subset of an n-ary semigroup A is a fuzzy n-ary subsemigroup but not an almost fuzzy n-ary subsemigroup of A. Next, let g be a nonzero fuzzy n-ary subsemigroup of an n-ary semigroup A. By Proposition 1, we have $\mathfrak{f}(g^n) \subseteq g$. This implies that $\mathfrak{f}(g^n) \cap g = \mathfrak{f}(g^n)$. Since $g \neq 0$, there exists $a \in A$ such that $g(a) \neq 0$. Then $\mathfrak{f}(g^n)(f(a^n)) \neq 0$. Hence $\mathfrak{f}(g^n) \cap g = \mathfrak{f}(g^n) \neq 0$. Therefore, g is a fuzzy almost n-ary subsemigroup of A. We can conclude that every nonzero fuzzy n-ary subsemigroup of an n-ary semigroup A is a fuzzy almost n-ary subsemigroup of A.

Example 2. We consider an n-ary semigroup $\mathbb{N}$ under the usual n-ary multiplication of integers. Let g and h be fuzzy subsets of $\mathbb{N}$ as follows:

$$g(a) = \begin{cases} 0.5 & \text{if } a = 2 \text{ or } 2^n, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$h(a) = \begin{cases} 0.4 & \text{if } a = 2^n \text{ or } 2^{n^2}, \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, g and h are fuzzy almost n-ary subsemigroups but are not fuzzy n-ary subsemigroups of $\mathbb{N}$. However, $g \cap h$ is not a fuzzy almost n-ary subsemigroup of $\mathbb{N}$.

From Example 2, we obtain the following conclusions.

(1) A fuzzy almost n-ary subsemigroup of an n-ary semigroup A need not be a fuzzy n-ary subsemigroup of A.

(2) The intersection of fuzzy almost n-ary subsemigroups of an n-ary semigroup A need not be a fuzzy almost n-ary subsemigroup of A.

Theorem 2. *Let g be a fuzzy almost n-ary subsemigroup of an n-ary semigroup A. If h is a fuzzy subset of A such that $g \subseteq h$, then h is also a fuzzy almost n-ary subsemigroup of A.*

Proof. Since g is a fuzzy almost n-ary subsemigroup of A, $\mathfrak{f}(g^n) \cap g \neq 0$. Since $g \subseteq h$, $\mathfrak{f}(g^n) \cap g \subseteq \mathfrak{f}(h^n) \cap h$. This implies that $\mathfrak{f}(h^n) \cap h \neq 0$. So the proof is completed.

As a direct consequence of Theorem 2, we obtain the following corollary.

Corollary 2. *The union of fuzzy almost n-ary subsemigroups of an n-ary semigroup A is also a fuzzy almost n-ary subsemigroup of A.*

3.3. The relationships between almost n-ary subsemigroups and their fuzzifications

This subsection is devoted to examining the relationships between almost n-ary subsemigroups and fuzzy almost n-ary subsemigroups of n-ary semigroups.

Theorem 3. *Let S be a subset of an n-ary semigroup A. Then S is an almost n-ary subsemigroup of A if and only if χ_S is a fuzzy almost n-ary subsemigroup of A.*

Proof. Assume that S is an almost n-ary subsemigroup of A. Then $S \neq \emptyset$ and $f(S^n) \cap S \neq \emptyset$. Hence, there exists an element a in S such that $a \in f(S^n) \cap S$, Therefore, $a = f(a_1^n)$ for some $a_1, a_2, \ldots, a_n \in S$ and $a \in S$. It follows that $\mathfrak{f}((\chi_S)^n)(a) = 1$ and $\chi_S(a) = 1$, which implies that $(\mathfrak{f}((\chi_S)^n) \cap \chi_S)(a) = 1 \neq 0$. We can conclude that $\mathfrak{f}((\chi_S)^n) \cap \chi_S \neq 0$. Hence, χ_S is a fuzzy almost n-ary subsemigroup of A. Conversely, assume that χ_S is a fuzzy almost n-ary subsemigroup of A. We have χ_S is a nonzero fuzzy subset of A and $\mathfrak{f}((\chi_S)^n) \cap \chi_S \neq 0$. Then there exists an element a of A such that $(\mathfrak{f}((\chi_S)^n) \cap \chi_S)(a) \neq 0$. So $\mathfrak{f}((\chi_S)^n)(a) \neq 0$ and $\chi_S(a) \neq 0$. This implies that $\mathfrak{f}((\chi_S)^n)(a) = 1$ and $\chi_S(a) = 1$. Hence, $a \in f(S^n)$ and $a \in S$. Eventually, $f(S^n) \cap S \neq \emptyset$. This concludes that S is an almost n-ary subsemigroup of A.

Theorem 4. *Let g be a fuzzy subset of an n-ary semigroup A. Then g is a fuzzy almost n-ary subsemigroup of A if and only if $supp(g)$ is an almost n-ary subsemigroup of A.*

Proof. Let g be a fuzzy almost n-ary subsemigroup of A. We have that $\mathfrak{f}(g^n) \cap g$ is not a zero fuzzy subset of A. Thus, there exists $a \in A$ such that $(\mathfrak{f}(g^n) \cap g)(a) \neq 0$. Then $g(a) \neq 0$ and $\mathfrak{f}(g^n)(a) \neq 0$. So $a = f(a_1^n)$ for some $a_1, a_2, \ldots, a_n \in A$ such that $g(a_i) \neq 0$ for all $i \in \{1, 2, \ldots, n\}$. This implies that $a_1, a_2, \ldots, a_n \in supp(g)$. Therefore $\mathfrak{f}((\chi_{supp(g)})^n)(a) \neq 0$ and $\chi_{supp(g)}(a) \neq 0$. Hence, $(\mathfrak{f}((\chi_{supp(g)})^n) \cap \chi_{supp(g)})(a) \neq 0$. So, $\chi_{supp(g)}$ is a fuzzy almost n-ary subsemigroup of A. By Theorem 3, $supp(g)$ is an almost n-ary subsemigroup of A. Conversely, assume that $supp(g)$ is an almost n-ary subsemigroup of A. By Theorem 3, we have $\chi_{supp(g)}$ is a fuzzy almost n-ary subsemigroup of A. Thus

$\mathfrak{f}((\chi_{supp(g)})^n) \cap \chi_{supp(g)} \neq 0$. Then there is $a \in A$ such that $(\mathfrak{f}((\chi_{supp(g)})^n) \cap \chi_{supp(g)})(a) \neq 0$. Hence, $\mathfrak{f}((\chi_{supp(g)})^n)(a) \neq 0$ and $\chi_{supp(g)}(a) \neq 0$. Then there exist $a_1, a_2, \ldots, a_n \in supp(g)$ and $a = f(a_1^n)$. Therefore $g(a_i) \neq 0$ for all $i \in \{1, 2, \ldots, n\}$. Hence, $\mathfrak{f}(g^n)(a) \neq 0$. This implies that $(\mathfrak{f}(g^n) \cap g)(a) \neq 0$. Consequently, g is a fuzzy almost n-ary subsemigroup of A.

An almost n-ary subsemigroup S of an n-ary semigroup A is called *minimal* if for any almost n-ary subsemigroup T of A such that $T \subseteq S$, it follows that $T = S$. Next, we examine the minimality of fuzzy almost n-ary subsemigroups. A fuzzy almost n-ary subsemigroup g of an n-ary semigroup A is called *minimal* if for any fuzzy almost n-ary subsemigroup h of A contained in g, it follows that $supp(g) = supp(h)$. Now, we provide the relationship between minimal almost n-ary subsemigroup and their fuzzifications.

Theorem 5. *A nonempty subset S of an n-ary semigroup A is a minimal almost n-ary subsemigroup of A if and only if χ_S is a minimal fuzzy almost n-ary subsemigroup of A.*

Proof. Let S be a minimal almost n-ary subsemigroup of an n-ary semigroup A. By Theorem 3, we have that χ_S is a fuzzy almost n-ary subsemigroup of A. Suppose that g is a fuzzy almost n-ary subsemigroup of A contained in χ_S. By Theorem 4, $supp(g)$ is an almost n-ary subsemigroup of A. Since $g \subseteq \chi_S$, $supp(g) \subseteq supp(\chi_S) = S$. Because S is minimal, we conclude that $supp(g) = S = supp(\chi_S)$. Therefore, χ_S is minimal. Conversely, suppose that χ_S is a minimal fuzzy almost n-ary subsemigroup of A, and let T be an almost n-ary subsemigroup of A contained in S. By Theorem 3, we have that χ_T is a fuzzy almost n-ary subsemigroup of A and $\chi_T \subseteq \chi_S$. Thus, $T = supp(\chi_T) = supp(\chi_S) = S$. We conclude that S is minimal.

Corollary 3. *An n-ary semigroup A has no proper almost n-ary subsemigroups if and only if for all fuzzy almost n-ary subsemigroup g of A, $supp(g) = A$.*

Proof. Assume that A has no proper almost n-ary subsemigroups, and let g be a fuzzy almost n-ary subsemigroup of A. By Theorem 4, we have $supp(g)$ is an almost n-ary subsemigroup of A. By assumption, we have $supp(g) = A$. To prove the converse, we let S be any almost n-ary subsemigroup of A. By Theorem 3, we have that χ_S is a fuzzy almost n-ary subsemigroup of A. By assumption, we get $S = supp(\chi_S) = A$. We can conclude that A has no proper almost n-ary subsemigroups.

Let A be an n-ary semigroup. An almost n-ary subsemigroup S of A is called *prime* if for all $a_1, a_2, \ldots, a_n \in A$, $f(a_1^n) \in S$ implies $a_i \in S$ for some $i \in \{1, 2, \ldots, n\}$. A fuzzy almost n-ary subsemigroup g of A is called *prime* if $g(f(a_1^n)) \leq \max\{g(a_1), g(a_2), \ldots, g(a_n)\}$ for all $a_1, g_2, \ldots, a_n \in A$. Next, we investigate a relationship between prime almost n-ary subsemigroups and their fuzzifications.

Theorem 6. *A nonempty subset S of an n-ary semigroup A is a prime almost n-ary subsemigroup of A if and only if χ_S is a prime fuzzy almost n-ary subsemigroup of A.*

Proof. Let S be any prime almost n-ary subsemigroup of A. By Theorem 3, we have that χ_S is a fuzzy almost n-ary subsemigroup of A. Let $a_1, a_2, \ldots, a_n$ be any n elements in A. If $f(a_1^n) \in S$, then $a_i \in S$ for some $i \in \{1, 2, \ldots, n\}$ because S is prime. Then $\chi_S(a_i) = 1$ for some $i \in \{1, 2, \ldots, n\}$. So $\chi_S(f(a_1^n)) \leq 1 = \max\{\chi_S(a_1), \chi_S(a_2), \ldots, \chi_S(a_n)\}$. If $f(a_1^n) \notin A$, then $\chi_S(f(a_1^n) = 0 \leq \max\{\chi_S(a_1), \chi_S(a_2), \ldots, \chi_S(a_n)\}$. By both cases, we can conclude that $\chi_S(f(a_1^n) \leq \max\{\chi_S(a_1), \chi_S(a_2), \ldots, \chi_S(a_n)\}$. Therefore, χ_S is a prime fuzzy almost n-ary subsemigroup of A. To prove the converse, suppose that χ_S is a prime fuzzy almost n-ary subsemigroup of A. By Theorem 3, we have that S is an almost n-ary semigroup of A. Let $a_1, a_2, \ldots, a_n$ be any n elements in A such that $f(a_1^n) \in S$. Thus, $\chi_S(f(a_1^n)) = 1$. By assumption, we have that $1 = \chi_S(f(a_1^n)) \leq \max\{\chi_S(a_1), \chi_S(a_2), \ldots, \chi_S(a_n)\}$. Hence, $\max\{\chi_S(a_1), \chi_S(a_2), \ldots, \chi_S(a_n)\} = 1$. We can conclude that $\chi_S(a_i) = 1$ for some $i \in \{1, 2, \ldots, n\}$. Thus $a_i \in S$ for some $i \in \{1, 2, \ldots, n\}$. Therefore, S is a prime almost n-ary subsemigroup of A.

Let A be an n-ary semigroup. An almost n-ary subsemigroup S of A is said to be *semiprime* if for all $a \in A$, $f(a^n) \in S$ implies $a \in S$. A fuzzy almost n-ary subsemigroup g of A is said to be *semiprime* if $g(f(a^n)) \leq g(a)$ for all $a \in A$. It is clear that every prime almost n-ary subsemigroup of A is semiprime. Similarly, every prime fuzzy almost n-ary subsemigroup of A is also semiprime. Finally, we present the relationship between semiprime almost n-ary subsemigroups and their fuzzifications.

Theorem 7. *A nonempty subset S of an n-ary semigroup A is a semiprime almost n-ary subsemigroup of A if and only if χ_S is a semiprime fuzzy almost n-ary subsemigroup of A.*

Proof. Let S be a semiprime almost n-ary subsemigroup of A. By Theorem 3, χ_S is a fuzzy almost n-ary subsemigroup of A. Let $a \in A$. If $f(a^n) \in S$, then $a \in S$ because S is semiprime. Thus $\chi_S(a) = 1$. Hence, $\chi_S(f(a^n)) \leq \chi_S(a)$. If $f(a^n) \notin S$, then $\chi_S(f(a^n)) = 0 \leq \chi_S(a)$. In both cases, we conclude that $\chi_S(f(a^n)) \leq \chi_S(a)$ for all $a \in A$. Therefore, χ_S is a semiprime fuzzy almost n-ary subsemigroup of A. Conversely, suppose that χ_S is a semiprime fuzzy n-ary subsemigroup of A. By Theorem 3, we have that S is an almost n-ary subsemigroup of A. Let a be an element in A such that $f(a^n) \in S$. Then $\chi_S(f(a^n)) = 1$. Since χ_S is semiprime, we have $\chi_S(f(a^n)) \leq \chi_S(a)$. It follows that $\chi_S(a) = 1$, and hence $a \in S$. Consequently, S is a semiprime almost n-ary subsemigroup of A.

4. Conclusion

In this paper, we introduce the notions of almost n-ary subsemigroups and fuzzy almost n-ary subsemigroups of n-ary semigroups. We prove that every n-ary semigroup is also an almost n-ary semigroup; however, the converse does not hold in general. Moreover, we show that the union of two almost n-ary subsemigroups is also an almost n-ary subsemigroup. However, the same does not generally hold for their intersection. Similarly, we have that the union of two fuzzy almost n-ary subsemigroups is also a fuzzy almost n-ary subsemigroup. Nevertheless, this is not generally true for their intersection. Furthermore, we present the relationships between almost n-ary subsemigroups and their corresponding fuzzifications (Theorem 3-7).

In future work, we aim to study other various types of ideals of n-ary semigroups and their corresponding fuzzifications.

Acknowledgements

The authors gratefully acknowledge the reviewers for their time, effort, and invaluable feedback. Their constructive critiques and thoughtful recommendations have played a vital role in refining the quality of this research.

References

[1] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–353, 1965.

[2] R. Kasner. An extension of the group concepts. *Bulletin of the American Mathematical Society*, 10:290–291, 1904.

[3] W. A. Dudek. Idempotents in n-ary semigroups. *Southeast Asian Bulletin of Mathematics*, 25:97–104, 2011.

[4] J. P. Solano; S. Suebsung and R. Chinram. On ideals of fuzzy points n-ary semigroups. *International Journal of Mathematics and Computer Science*, 13(2):179–186, 2018.

[5] M. Couceiro and J. Devillet. Every quasitrivial n-ary semigroup is reducible to a semigroup. *Algebra Universalis*, 80(4):51, 2019.

[6] C. Somsup and U. Leerawat. Congruences and homomorphisms on n-ary semigroups. *International Journal of Mathematics and Computer Science*, 15(2):671–682, 2020.

[7] P. Pornsurat and B. Pibaljommee. Left regular and left weakly regular n-ary semigroups. *Kyungpook Mathematical Journal*, 62(1):29–41, 2022.

[8] M. Couceiro; J. Devillet; J. L. Marichal and P. Mathonet. Reducibility of n-ary semigroups: from quasitriviality towards idempotency. *Beitrage zur Algebra und Geometrie*, 63(1):149–166, 2022.

[9] J. Daengsaen and S. Leeratanavalee. Semilattices of simple and regular n-ary semigroups. *Semigroup Forum*, 107(2):294–314, 2023.

[10] O. Grosek and L. K. Satko. A new notion in the theory of semigroup. *Semigroup Forum*, 21(5):233–240, 1980.

[11] K. Wattanatripop; R. Chinram and T. Changphas. Quasi-A-ideals and fuzzy A-ideals in semigroups. *Journal of Discrete Mathematical Sciences and Cryptography*, 21(5):1131–1138, 2018.

[12] P. Khamrot and T. Gaketem. Applications of bipolar fuzzy almost ideals in semigroups. *International Journal of Analysis and Applications*, 22:art. no. 8, 2024.

[13] P. Khamrot and T. Gaketem. On picture fuzzy almost ideals of semigroups. *Journal of Discrete Mathematical Sciences and Cryptography*, 27(6):1817–1830, 2024.

[14] A. Iampan; R. Chinram and P. Petchkaew. A note on almost subsemigroups of semi-

groups. *International Journal of Mathematics and Computer Science*, 16(4):1623–1629, 2021.

[15] R. Chinram; S. Kaewchay; A. Iampan and P. Singavananda. Characterization of almost ternary subsemigroups and their fuzzifications. *Journal of Mathematics and Computer Science*, 27(2):97–104, 2022.

[16] R. Rittichuai; A. Iampan; R. Chinram and P. Singavananda. Almost subsemirings and fuzzifications. *International Journal of Mathematics and Computer Science*, 17(4):1491–1497, 2022.

[17] N. Sarasit; R. Chinram and A. Rattana. Applications of fuzzy set for almostity of ternary subsemirings. *International Journal of Applied Mathematics*, 36(4):497–508, 2023.

[18] P. Petchkaew and R. Chinram. The minimality and maximality of n-ideals in n-ary semigroups. *European Journal of Pure and Applied Mathematics*, 11(3):762–773, 2018.

[19] J. N. Mordeson; D. S. Malik and N. Kuroki. Fuzzy semigroups. *Springer-Verlag, Berlin*, 2012.